

Resolution of an Inconsistency in Deep Space Station Longitude Solutions

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This article presents analysis and results that lead to the resolution of a discrepancy in Deep Space Station (DSS) longitude estimates that had been obtained in 1971 and 1972 from spacecraft near-encounter radio metric data. A 21-m discrepancy between the Mariner 4 and Mariner 9 DSS longitude solutions is shown to be reduced to within 3 m with the application of improved solution strategies. The resulting agreement between all encounter arc longitude solutions for Mariners 4, 5, 6, and 11 is within 5 m.

I. Introduction

In 1971, during the preparation of DSS location estimates for the Mariner 9 navigation operations, there appeared a large discrepancy between the three distinct longitude estimates that were obtained from processing the Mariner 4, 5 and 6 near encounter data sets (Ref. 1). Specifically, the DSS longitudes that were determined from the Mariner 4 near-encounter data were consistently removed from the Mariner 5- and 6-based solutions, by as much as 14 m, whereas the Mariner 5 and 6 solutions agree to within 1 m. The processing of the subsequently obtained Mariner 9 near-encounter data, instead of clarifying matters, produced DSS longitude solutions that were 7 to 8 m from the Mariner 5 and 6 values in a direction opposite to the Mariner 4 displacement and,

hence, roughly 21 m removed from the Mariner 4 determination. A disagreement of this size, of course, shed considerable doubt on the reliability of the current DSS location determination process required by the DSN in its support of the interplanetary flight projects. When these results persisted in the face of concerted analysis a special study team was established in April 1973 to resolve what had then become known as "the longitude problem."

The resolution of the longitude problem was achieved in October 1973. Basically it was found that the Mariner 4 and 9 determinations were subject to a solution instability due to insufficient postencounter radio data coverage. Mariner 9 was, of course, tracked postencounter as it successfully inserted into Mars orbit. The large inser-

tion burn, however, precluded the useful processing of the postencounter data. Mariner 4 was tracked post-encounter, although only a relatively small amount of two-way Doppler data were obtained due to required picture playback activity.

With the identification of the possible data deficiencies, strengthened determination strategies could be specified for both data sets. The measures that were applied did, indeed, improve the Mariner 4/Mariner 9 agreement, to a value less than 3 m. The resulting agreement for all the available encounter data sets, Mariners 4, 5, 6 and 9 was reduced to within ± 2.5 m.

This article presents the supporting analysis and arguments for the October 1973 longitude problem resolution. This material, however, includes only a part of the entire longitude problem effort and cannot be taken as a summarization of all the effort that contributed to the resolution.¹ Much of the work has unfortunately gone unpublished, principally due to the negative nature of most of the results. These results allowed, nevertheless, a focusing of the effort into the areas that finally produced the resolution.

The contents of this article are organized as follows: In the following two sections the basic characteristics of the longitude problem are described, and the general problem of determining DSS locations from encounter radio metric data is discussed. The role of spacecraft geocentric range rate uncertainty in the determination of station location solutions is introduced as a possible source of solution instability.

In the next sections the Mariner 4 and 9 near-encounter data sets are analyzed. The Mariner 4 solutions are shown to give better agreement with the other encounter solutions as Doppler measurements are added to the conventionally used, encounter ± 5 -day data set. The credibility of the extended data arc solutions is established by sensitivity analysis. The effect of spacecraft geocentric range rate accuracy on longitude solutions is explicitly shown for the Mariner 9 data set with the use of near-encounter range measurements. A special processing of these measurements is shown to improve the longitude solution agreement by 2 to 3 m. With these adjustments to the Mariner 4 and 9 solutions, the agreement between all the Mariner encounter solutions im-

proves to within ± 2.5 m. This range is considered consistent with the expected error in the individual longitude determinations.

II. Character of the Longitude Problem

Figure 1 shows the preresolution state of the DSS longitude disagreement. The station coordinates are shown in terms of spin-axis distance and longitude for each of the encounter data sets, Mariners 4, 5, 6, and 9. The solutions are based on the DE84 ephemeris and are referenced to a post-Mariner 9 encounter station location set, LS37. LS37 is a minor update to the location set LS35 described in Ref. 1.

The Mariner 4/Mariner 9 disagreement is shown to be 14.5 m, a reduction by 6.5 m from the 21-m disagreement that was understood to be the status of the longitude problem in April 1973. The reduction was due to two factors:

- (1) A 2.0- to 2.5-m reduction in the Mariner 4 displacement from LS37 when the ephemeris reference was shifted from DE78 to DE84.
- (2) The establishment of the Mariner 9 solutions on a consistent ephemeris reference with respect to the Mariner 4, 5 and 6 solutions. As indicated, the reference ephemeris for the solutions shown in Fig. 1 is DE84. When the longitude problem was first identified the solutions were, by error, not consistently referenced: the Mariner 9 solutions were referenced to DE80 whereas the other solutions were referenced to DE78. Thus, approximately 4 m of the 21-m disagreement were nonexistent.

The Mariner 4/Mariner 9 separation is still shown to be an unacceptably large 14 to 15 m. The relative longitudes and spin axis values, however, show good agreement. This fact had been taken as an indication that the longitude problem was due to error in the determinations of the rotation of the Earth, the precession of the equinox, or possible drifts in the planetary ephemeris. Investigations carried out in these areas, however, indicated that the source of the longitude problem should be sought elsewhere.

The investigation was thus narrowed to consider the principal remaining error source—that of the actual station location estimation process—and specifically the

¹The principal contributors to the effort were D. W. Trask, J. Ellis, H. F. Fliegel, J. F. Jordan, T. Nishimura, S. J. Reinbold, H. N. Royden, M. A. Slade, M. Standish, F. B. Winn, and J. W. Zielenbach.

estimation effects that produce large absolute longitude errors, yet allow the corresponding relative longitude and spin axis values to be well behaved.

III. Conceptual Analysis of Station Location Determination

In this section the determination of DSS locations is considered from the basic Hamilton-Melbourne (Ref. 2) point of view. It is shown that a mechanism exists that can explain the above-stated characteristics of the Mariner 4/Mariner 9 disagreement; specifically, because of uncertainties in the spacecraft range rate, it is shown that station absolute longitude estimates are more sensitive to data errors than spin axis or relative longitude estimates. This point serves as a motivation for the analysis of the Mariner 4 and Mariner 9 encounter data sets presented in the succeeding sections. The value of the approach is not necessarily compelling in the case of Mariner 4 although the longitude estimates do show better agreement if larger data sets are used. The value of the approach is clear from the analysis of the Mariner 9 data set, however, as the treatment of geocentric range rate uncertainties is shown to improve both the longitude agreement and the formal solution accuracies as well.

Figure 2 illustrates the basic geometry of station location determination using encounter radio data, with emphasis on longitude related parameters. Hamilton and Melbourne's well-traveled analysis shows that a very good and instructive approximation to the range rate or Doppler observable can be expressed as follows:

$$\dot{\rho} = \dot{r} + r_s \omega \cos \delta \sin [\alpha_{GR}(t) - \alpha_{S/C}(t) + \lambda]$$

where

$\dot{\rho}$ = range rate observable

\dot{r} = geocentric range rate of spacecraft

r_s = DSS distance from the Earth spin axis

ω = Earth rotation rate

δ = spacecraft declination

$\alpha_{GR}(t)$ = right ascension of Greenwich

$\alpha_{S/C}(t)$ = right ascension of spacecraft

A convenient time reference for this representation is the nominal time of the spacecraft crossing of the local nomi-

nal DSS meridian, i.e., t_0 such that

$$\alpha_{GR}(t_0) + \lambda_0 = \alpha_{S/C}(t_0)$$

where λ_0 is the nominal DSS longitude.

The above expression then reduces to

$$\dot{\rho} = \dot{r} + r_s \omega \cos \delta \sin [\omega(t - t_0) - \Delta\alpha_{S/C} + \Delta\lambda]$$

with $\pi/2 \leq (t - t_0) \pi/2$, approximately, for a single day's pass, where $\Delta\lambda = \lambda - \lambda_0$ and

$$\Delta\alpha_{S/C} = \alpha_{S/C}(t_0) - \alpha_{S/C_0}(t_0)$$

for α_{S/C_0} , the nominal spacecraft right ascension at t_0 . The value of the near-encounter radio data is that the spacecraft position coordinates, i.e., α and δ , are accurately determined relative to the encounter planet position. A planetary ephemeris then provides an absolute reference for the spacecraft position near the time of encounter e.g., encounter ± 5 days. Thus assuming no difficulties with the planetary ephemeris or timing standards, the Doppler observable expressed in terms of the remaining uncertain parameters \dot{r} , r_s and $\Delta\lambda$ can be given as follows:

$$\dot{\rho} = \dot{r} + r_s \omega \cos \delta \cos (\omega t + \Delta\lambda)$$

letting $t_0 = 0$.

One may question the addition of \dot{r} , the spacecraft geocentric range rate as an uncertainty to be considered along with station longitude and spin axis errors. This is, however, just the point that is to be established: that although spacecraft range rate is well observed by Doppler observations, only small uncertainties in this parameter can still degrade the ability to accurately determine DSS locations. To illustrate this, the observable equation can be used to obtain the following expression for the variation in range rate as a function of variations in \dot{r} , r_s , and $\Delta\lambda$:

$$\Delta\dot{\rho} = \Delta\dot{r} + (\omega \cos \delta \sin \omega t) \Delta r_s - (r_s \omega \cos \delta \cos \omega t) \Delta\lambda$$

Thus, as is shown in Fig. 3, incremental effects in observed range rate take the form of a bias for $\Delta\dot{r}$, a sine curve for Δr_s , and a cosine curve for $\Delta\lambda$. The curves in Fig. 3 serve to illustrate that the $\Delta\dot{r}$ and $\Delta\lambda$ range rate effects are somewhat similar in appearance in that they are even functions about $t = 0$ in contrast to the odd function nature of the Δr_s effect. Due to this similarity, geocentric range rate and longitude are relatively difficult to independently extract from a solution based on a single

pass of DSS data. This fact is pointed out in Ref. 2 and is illustrated quite well in Fig. 4, which is taken from that reference. Note that for increasing pass half widths, down to a practically used 75 deg² that $\Delta\dot{r}$ and $\Delta\lambda$ estimation accuracies, given 1 mm/s observation noise, are strongly linked and only approach spin-axis accuracies for low tracking elevations. The accuracy of $\Delta\lambda$ is seen to improve considerably if \dot{r} is perfectly known, i.e., if the spacecraft range rate is fixed or very accurately determined by an alternate information source. This behavior is the result of a high correlation between the $\Delta\lambda$ and $\Delta\dot{r}$ estimation errors, 0.96 for 75 deg half-pass widths, which is due to the similarity of the longitude and range rate effects illustrated in Fig. 3. As long as this correlation is high, DSN longitude can be determined only to the extent that geocentric range rate is known. The correspondence can be expressed approximately as

0.1 mm/s range rate accuracy \sim 1.4 m longitude accuracy

i. e., $\Delta\dot{r} \sim \omega\Delta\lambda$.

The accuracies shown in Fig. 4 are obtained from a formal error analysis of the Hamilton-Melbourne Doppler representation. Formal accuracy analysis is notoriously optimistic, and hence the actual numerical results should be considered carefully. What is of use for this discussion is the relative behavior of these accuracies. The actual values of 1 and 2 m indicated in Fig. 4 as, respectfully, spin axis and longitude accuracies are not of particular interest, since these values are directly proportional to a rather arbitrarily set 1 mm/s 1-min data noise standard deviation. Nevertheless, the fact that station longitude is relatively less well determined than station spin axis and that station longitude accuracy depends heavily on the observability of spacecraft geocentric range rate can be considered as fundamental to the method of determining DSS locations using short arcs of Doppler data.

The conclusions regarding longitude and spacecraft range rate correlation can be extended to multiple passes and additional stations. Combining multiple passes alone will not affect the correlation; they will only reduce the error by a $1/\sqrt{N}$ factor. Additional stations reduce the correlation between each individual station longitude error and the spacecraft geocentric range rate, but not by a large amount. One can show, in fact, that for N individual station passes the Hamilton-Melbourne model produces a correlation between individual station longi-

tudes and geocentric range rate that can be expressed as

$$\frac{\rho}{\sqrt{(N-1)(1-\rho^2)} + 1}$$

where ρ is the individual pass λ, \dot{r} correlation. The mutual correlation for longitudes of stations DSS_i DSS_j is then

$$\rho_{\lambda_i\lambda_j} = \frac{\rho^2}{(N-1)(1-\rho^2) + 1}$$

Assuming $N = 3$ and $\rho = 0.96$, for example, $\rho_{\lambda_i\lambda_j}$ is still large at a value of 0.80. Hence, each of the longitude accuracies is still principally dependent on the geocentric range rate accuracy. Thus, in general, the proposed mechanism, that is, the effect of uncertain spacecraft geocentric range rate on longitude estimates provides for the observed attributes of the longitude problem. That is, the longitude instability is correlated from station to station, while the relative longitudes and spin-axis distances remain unaffected.

It was mentioned previously that longitude stability is improved if spacecraft geocentric range rate information is somehow strengthened. Such an effect can, of course, be expected for data sets including a spacecraft encounter since near-encounter trajectory bending does provide a complete orbit estimate virtually independent of Earth-based reference parameters such as station locations. Thus accurate station location estimates are possible from near-encounter data because of the accurate spacecraft velocity as well as the accurate spacecraft position determinations afforded by close-encounter radio measurements. This effect serves as an explanation for the apparent instability of and poor agreement between the Mariner 4 and Mariner 9 encounter DSS location determinations. As mentioned in the following sections these missions have incomplete metric tracking coverage post-encounter, possibly indicating degraded encounter orbit velocity estimates. In the case of Mariner 4 this is not directly shown; however, for Mariner 9 it is clearly demonstrated that an improved spacecraft velocity determination does indeed improve the agreement and hence the apparent accuracy of the DSS location estimates.

IV. Mariner 4 Encounter Data Analysis

The Mariner 4 near-encounter tracking coverage is shown in Fig. 5 in terms of station elevation profiles. The time period shown is Mars encounter, July 15, 1965, ± 5 days—the traditional data arc for determining DSS

²Minimum elevations around 15 deg are usually employed to limit atmosphere effects.

locations. As indicated, DSS 11 (Goldstone), DSS 42 (Canberra) and DSS 51 (Johannesburg) tracked Mariner 4 during this period. The coverage is seen to be nearly continuous before encounter but is sparse shortly after. This is due to picture playback activity which required reduced two-way Doppler tracking after encounter. In light of previous discussion the meager postencounter coverage of Mariner 4 possibly indicates that the encounter ± 5 -day data set may be inadequate in producing reliable location estimates.

Figure 6 summarizes various treatments of the encounter ± 5 -day data set. Longitude and spin-axis estimates are shown for two data sets: one, including all the available data, and the other including the available data for which charged particle calibrations³ exist. The difference between these two sets is shown in Fig. 5. For the latter data set, station location solutions are shown with and without calibrations. Solutions are also shown when the GM and J_2 of Mars is estimated along with station locations and spacecraft state. It is clear from these results that the prime cause of longitude solution variation is variation in data set and not calibrations or solution vector choice. Note that spin-axis variations and relative longitude variations are less than the absolute longitude variations. The corresponding solutions for spacecraft (B plane) position are known to be essentially invariant, i.e., only sub-kilometer variations are observed. At the Mariner 4 Earth-Mars encounter distance of 200 million kilometers, the geometric interpretation of a 1-m longitude shift would imply a 42-km spacecraft position change. This indicates that the longitude solution variations are due to a more subtle error type, such as the effect of spacecraft geocentric range rate described earlier.

The possibility of solution instability due to insufficient coverage can be investigated by simply including additional data. This effect is illustrated in Figs. 7 and 8 which show station longitude change as a function of additional radio data included past Mars encounter. Indicated in each figure are the data intervals for which longitude solutions are plotted. In Fig. 7 the data set spans from encounter -5 days to encounter $+5$ days, and in Fig. 8 the data set spans from encounter -15 days to encounter $+15$ days. In both cases only spacecraft state and station location parameters are solved for. No calibrations are added for either case, since data for which calibrations are available are only contained in the encounter ± 5 -day interval. The calibration effect for the additional data can

be assumed to be similar to the slight variations for the encounter ± 5 -day calibrations. The additional 20 days of radio data is seen to produce marked improvement in the Mariner 4 agreement with LS 37. Improvement occurs right after encounter and holds up throughout the remainder of the data set.

The station location solutions for the two data sets are summarized in Table 1. It is of interest that the spin axis values and relative longitudes vary by no more than 1 and 2 m, respectively, although the longitude values move by as much as 4 m.

The principal concerns accompanying a longer arc determination of station locations can be given as follows:

- (1) The spacecraft position reference will become sun-centered rather than planet-centered and in the event of a planet right ascension error cause a shift in DSS longitudes.
- (2) Spacecraft non-gravitational accelerations will corrupt the radio measurements to greater extent over a longer arc and hence cause erroneous variations in the DSS location estimates.

Figure 9 presents the results of analysis directed at testing the validity of the longer arc longitude estimates. The general stability of longer arc longitude estimates is shown in Fig. 9a. The solution value is seen to decrease sharply between the 10- and 30-day arcs and the value remains relatively stable for up to 90-day arcs, i.e., encounter ± 45 days. Figure 9b addresses the above-stated concerns regarding the reliability of longer arc DSS location solutions. As shown, the stability of the Mars right ascension perturbation on the longitude solutions demonstrates that the location determinations remain "Mars referenced" for arcs up to 70 days in length. The perturbation is, in fact, closely approximated by a value of 2.4 m, predicted by simplified geometric reasoning. The effect of a nominal 10^{-12} km/sec² (bias) nongravitational acceleration perturbation also demonstrates that the longer encounter arc solutions are not unduly sensitive to spacecraft acceleration uncertainty.

In summary, the Mariner 4 solutions are seen to be unstable for the traditional encounter ± 5 day arc. Longer arcs about encounter exhibit improved stability and better agreement with the reference station location set and, hence, the other Mariner encounter station location determinations. These results are not entirely satisfying, since no explicit cause for the longitude "errors" is found, only implications regarding the cause of the improved

³The calibrations were prepared by K. W. Yip, JPL Section 391, and are based on Faraday rotation and ionosonde measurements.

agreement can be made. Nevertheless large longitude variations are observed as a function of solution strategy, and those estimates that best agree with the other Mariner values exhibit more stability than those that do not.

V. Mariner 9 Encounter Data Analysis

The Mariner 9 spacecraft inserted into Mars orbit shortly after 0 hours GMT on November 14, 1971. Because of the large ΔV required for orbit insertion, the postencounter radio data cannot be combined with pre-encounter data for improving station location estimates. Thus, as in the case of Mariner 4, the Mariner 9 longitude solutions may be somewhat unstable due to an imprecise determination of the entire orbit state vector, including position and velocity.

That such an instability does occur is shown quite clearly in Fig. 10. The station longitude solutions are plotted again as functions of increasing data arc length up to the last of the usable preencounter radio data at approximately Mars orbit insertion minus 30 min; the data arc begins at 5 days before encounter on November 9. The longitude solutions are seen to vary considerably, by as much as 10 m in the last 12 hours before encounter. This behavior is not at all inconsistent with the longitude formal error also plotted in Fig. 10. Figure 11 presents an equivalent history for the station spin-axis estimates. Their values are considerably more stable, which suggests that the longitude instability is due to spacecraft geocentric range rate uncertainty as proposed.

According to the range rate uncertainty hypothesis, an improved, independent range rate determination will improve the accuracy of station longitude determination. Unlike Mariner 4 this is indeed possible for Mariner 9, since the Mariner 9 spacecraft had a ranging transponder, and range measurements were taken near Mars encounter. Since range measured over time determines the change in range, a determination of mean range rate can be obtained to almost any accuracy using range measurements over a sufficiently long time interval.

The value of this approach depends on the inherent quality of the range measurements, and more specifically the stability of ranging delay over several days. The range quality can be evaluated with orbital data fit residuals. Shown in Fig. 12 are a set of DSS 14 MU range residuals referred to a Doppler-only orbit based on a data arc extending from encounter minus five days to encounter—the same arc used for the solutions shown in

Figs. 10 and 11. In using a doppler-only fit the range measurements can be evaluated against a reference that is not itself affected by the range measurements. The data shown in Fig. 12 are remarkable from two points of view. First, the range residuals exhibit a marked “ramp” indicating a significant range rate bias in the Doppler-only orbit. Secondly, accounting for the slope in the range, the range residuals show very good internal consistency, to within 10 m or better. This result does not indicate that something is particularly wrong with the Doppler orbit; it principally shows the strength of accurate range measurements in determining *mean* range rate. Indeed, 10-m ranging accuracy implies a mean range rate determination accuracy of roughly

$$\frac{10 \text{ m}}{3 \cdot 86400 \text{ s}} \sim 0.04 \text{ mm/s}$$

which is superior to the ~ 0.1 mm/s range-rate accuracy obtainable from 1 mm/s 1-min Doppler taken over 5 days.

Based on earlier analysis, the range rate error implied in Fig. 12 indicates that a shift in DSS longitude solutions will occur if range data are included with the Doppler. The expected shift is on the order of

$$(14 \text{ m/mm/s}) \cdot (0.36 \text{ mm/s}) = 5 \text{ m}$$

The extent of the effect that the range data have on the Doppler plus range solution depends on the assigned range data weight vis-à-vis the Doppler data weight. As indicated, a range weight that corresponds to an assumed 10-m range accuracy should be sufficient to assure that the range measurements control the range rate determination in the combined data fit. In addition to assigning proper data weights, special care should be taken when combining the range and Doppler data types in alleviating possible conflicting effects due to biases in the range measurements. Biases will not affect a range rate determination unless they are not properly accounted for, e.g., included as solve-for parameters. Biases can arise from instrumentation uncertainties or possible errors in the Earth-Mars distance as specified by the ephemeris and can be particularly troublesome, if unaccounted for, whenever very accurate, e.g., 10-m, range accuracy assumptions are used.

The notions concerning the use of range measurements have been applied to the Mariner 9 5-day preencounter data arc; the results are shown in Figs. 13 and 14, which

correspond to Figs. 10 and 11, which show Doppler-only solution variations. The results shown in Fig. 13 are striking in three particular ways:

- (1) The longitude solution movement is considerably more stable than the corresponding variations shown in Fig. 10.
- (2) The final solution values are shifted upward by 3 m, thereby producing better agreement with LS37.
- (3) The formal error is reduced and decreases much less drastically near encounter.

Figures 13 and 14 also show that the relative longitude and spin axis solutions are not largely affected by the addition of the range measurements, which is consistent with the properties of the range-rate effect hypothesis. The spin-axis values are seen to be biased below the LS37 reference. This reflects the absence of the ionosphere calibrations which invariably shift spin-axis solutions upward approximately 2 to 3 m for this particular encounter geometry.

These results clearly show the effect of spacecraft geocentric range rate accuracy on the determination of station longitudes. In addition, the value of range measurements in estimating DSS station locations is demonstrated. The described techniques for incorporating range data into the station location solutions should be applicable and beneficial in determining not only station locations but also spacecraft orbits in general, whenever sufficiently accurate range data are available.

VI. Final Summary

The modifications of the Mariner 4 and Mariner 9 longitude solutions that are described in this article are considered to constitute the resolution of the longitude problem. The resulting status of the individual mission DSS location solutions is shown in Fig. 15. The total longitude spread is no more than 5 m and Mariner 4 and Mariner 9 solutions agree to within 2 m.

New estimates are included for the Mariner 6 data set that show the effect of applying the range measurements according to the strategy employed for Mariner 9. A comparison of Figs. 1 and 15 reveals that, as in the case of Mariner 9, properly applied range data have improved the agreement of the Mariner 6 estimates with LS37 and the other Mariner estimates as well. Recent analysis of the Mariner 5 data set, however, has not indicated any improvement in the DSS location estimates when range measurements are used. Hence, no changes in the Mariner 5 estimates are shown. All of the estimates shown in Fig. 15 have been incorporated into a latest update of the DSS location estimates, denoted LS41. This new location set is described briefly in Ref. 3.

It is seen, therefore, that LS37, which is essentially based on the Mariner 5/Mariner 6 encounter solutions, proves to be a good determination of the DSS locations. The disagreement between the Mariner 4 and the Mariner 9 longitude estimates has been shown to arise from a basic instability in these estimates. The obtained improvements in the Mariner 4/Mariner 9 solution stabilities therefore only enhances the reliability of the already accurate DSS location determinations.

References

1. Hamilton, T. W., and Melbourne, W. G., "Information Content of a Single Pass of Doppler Data From a Distant Spacecraft," in *The Deep Space Network*, Space Programs Summary 37-39, Vol. III, pp. 18-23, Jet Propulsion Laboratory, Pasadena, Calif., May 31, 1966.
2. Madrid, G. A., et al., *Tracking System Analytic Calibration Activities for the Mariner Mars 1971 Mission*, Technical Report 32-1587, Jet Propulsion Laboratory, Pasadena, Calif., March 1, 1974.
3. Rourke, K. H., and Mottinger, N. A., "LS41—Final Pre-Venus M-10 Station Location Update," Interoffice Memorandum 391.8-147, Jan. 15, 1974 (JPL internal document).

Table 1. Mariner 4 DSS location solutions

Arc	DSS	Longitude, m		Spin axis, m	
		$\lambda - \lambda_{LS37}$	σ_λ	$r_s - r_{s,LS37}$	σ_{r_s}
E - 5 day \rightarrow E + 5 day	11	7.7	1.8	-0.9	0.8
	42	6.3	1.7	0.3	0.8
	51	5.4	1.5	1.0	0.9
E - 15 day \rightarrow E + 15 day	11	3.3	1.2	-1.5	0.6
	42	2.8	1.1	0.2	0.6
	51	3.1	1.1	-0.2	0.6

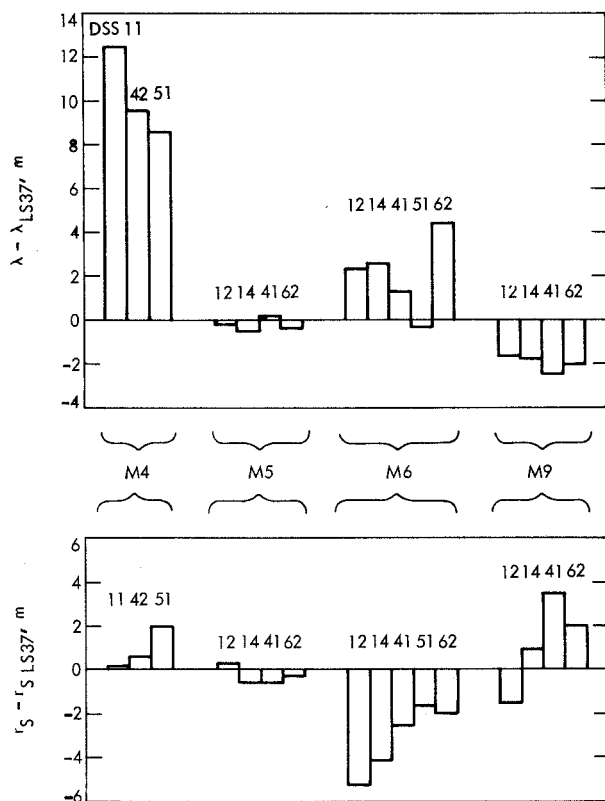


Fig. 1. Preresolution DSS location estimates

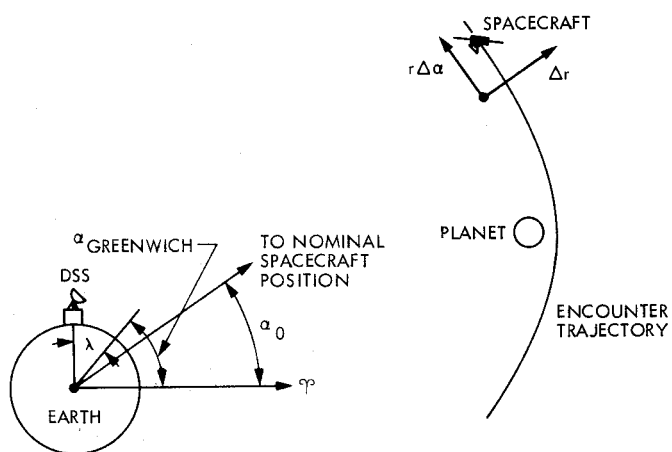


Fig. 2. Encounter geometry

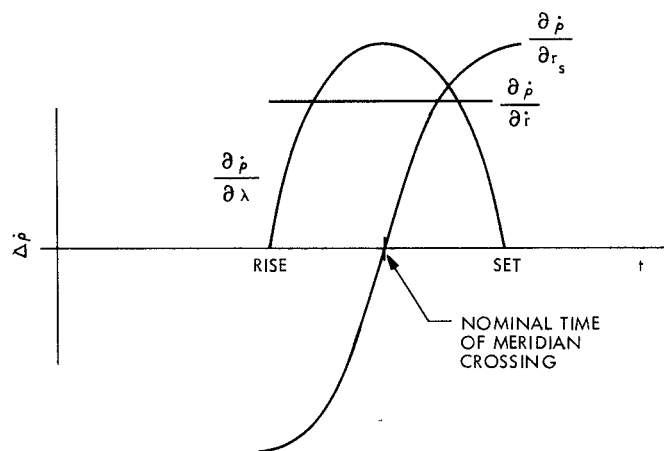


Fig. 3. Three-parameter analysis

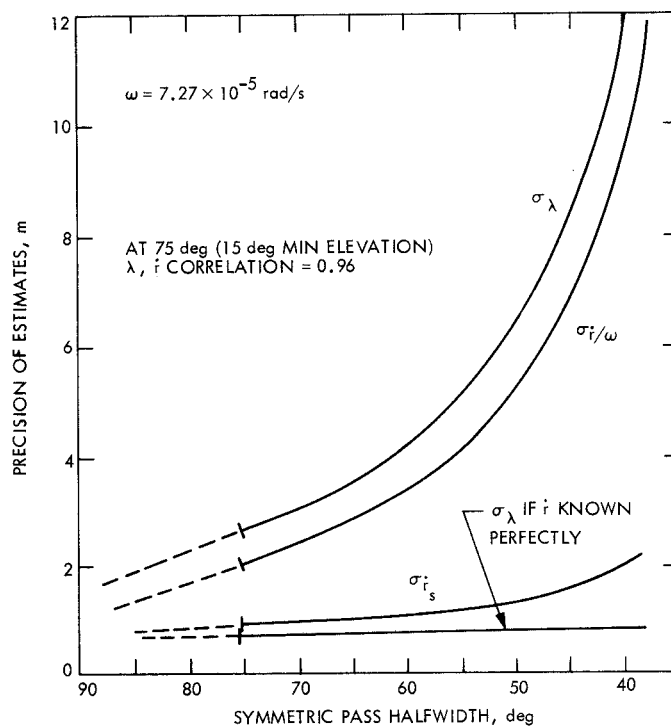


Fig. 4. Idealized one-pass accuracy

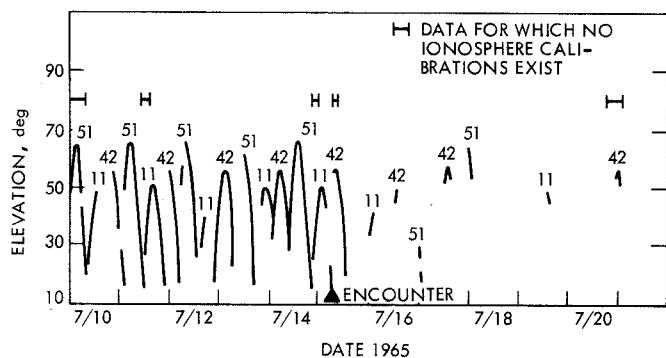


Fig. 5. Station coverage for Mariner 4 encounter

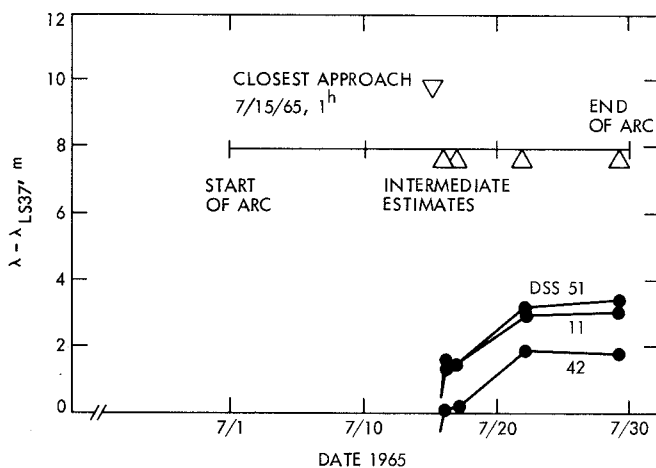


Fig. 7. Mariner 4 longitudes (data from 7-10-65)

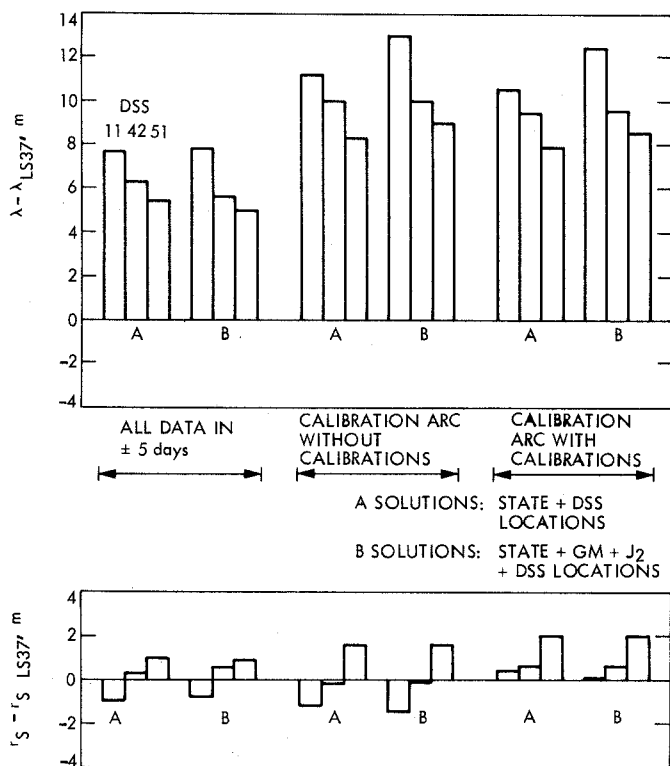


Fig. 6. Treatment variations for the Mariner 4 near-encounter data arc

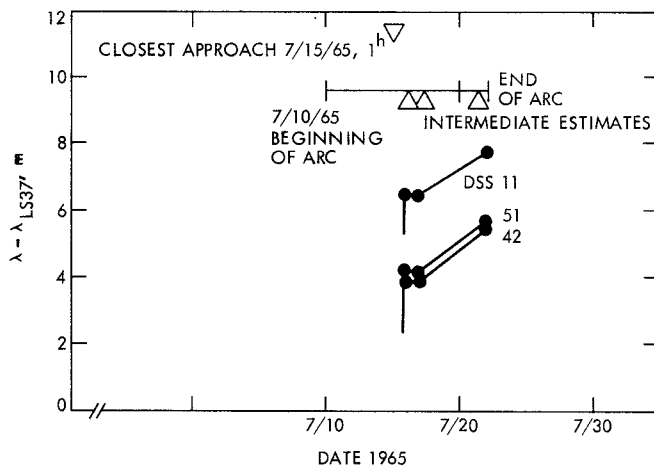


Fig. 8. Mariner 4 longitudes (data from 7-1-65)

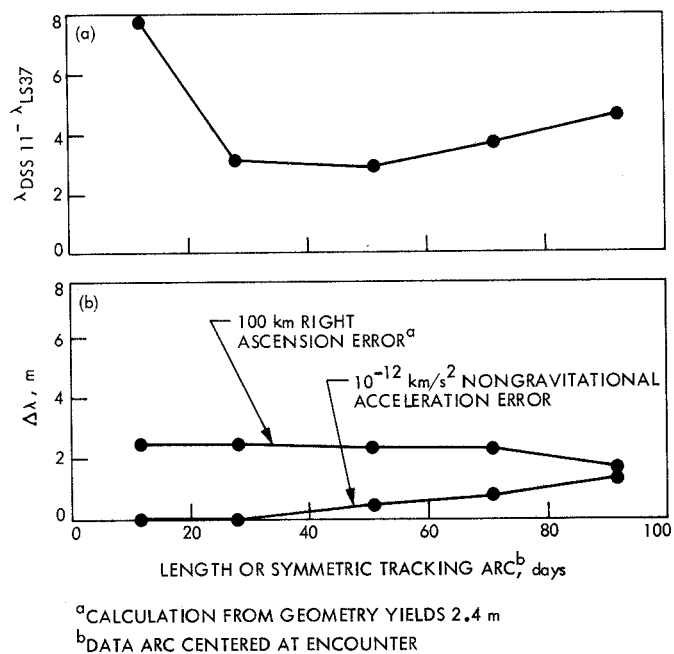


Fig. 9. Sensitivities for symmetric tracking arc: (a) Goldstone DSSC Mariner 4 longitude, (b) longitude perturbations due to Mars right ascension and spacecraft nongravitational acceleration

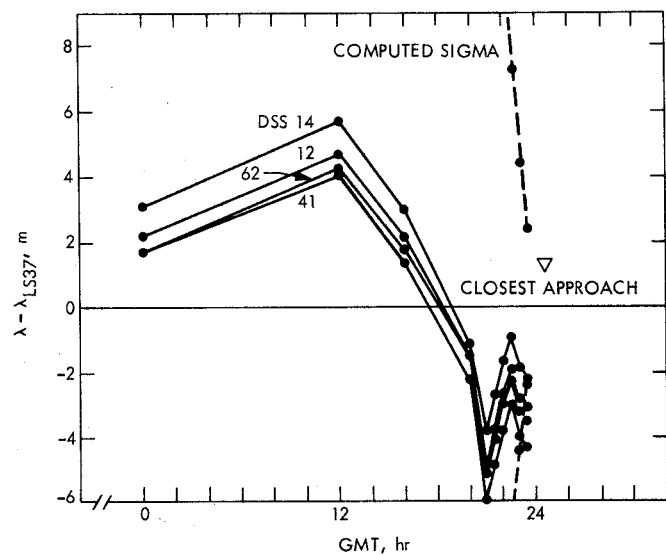


Fig. 10. Mariner 9 longitudes on 11-13-71 (Doppler only)

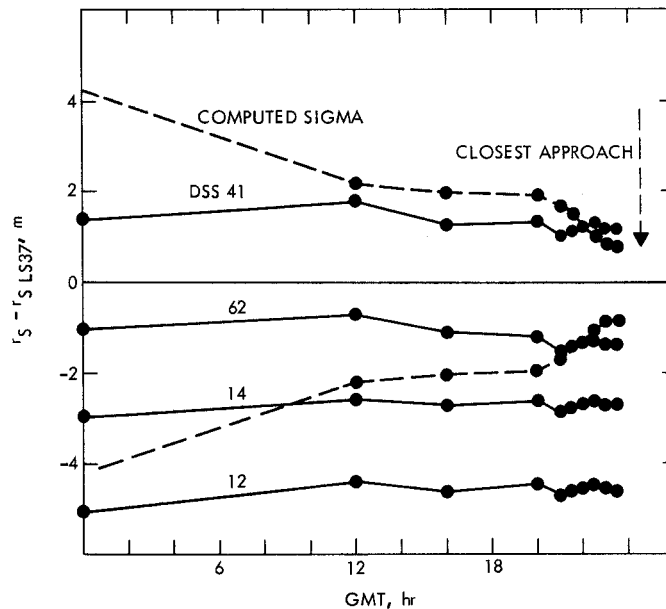


Fig. 11. Mariner 9 spin axis values on 11-13-71 uncalibrated (effect of calibrations bias solutions ~2 m positive) Doppler

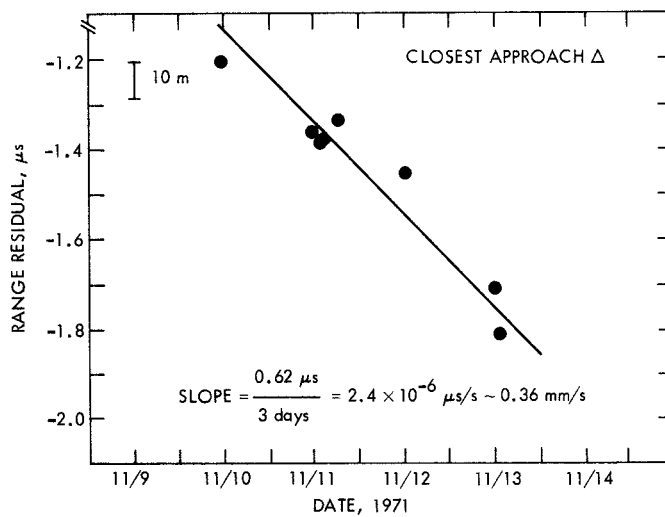


Fig. 12. Mu range with respect to encounter arc Doppler-only fit

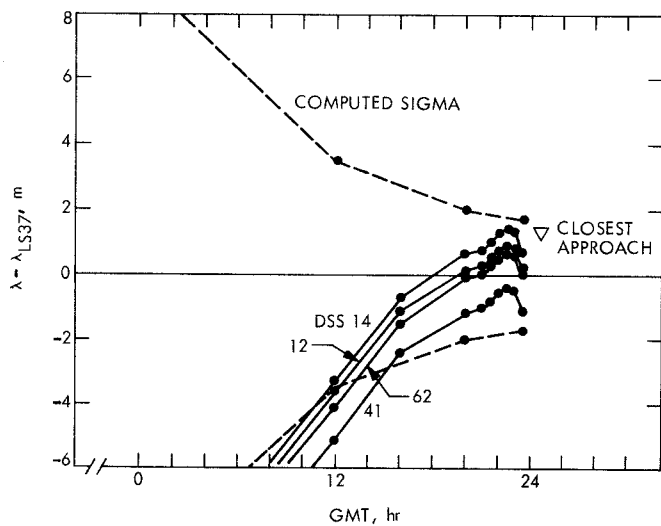


Fig. 13. Mariner 9 longitudes on 11-13-71 Doppler plus range

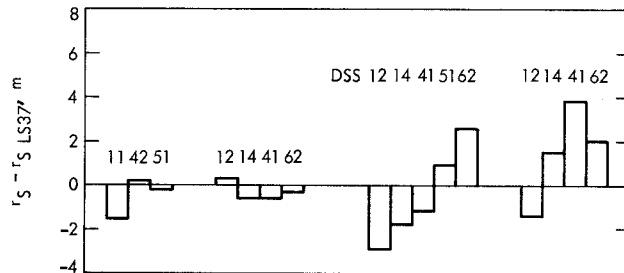
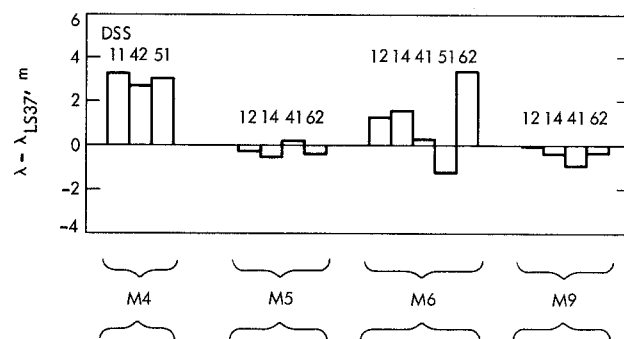


Fig. 15. Postresolution DSS location estimates

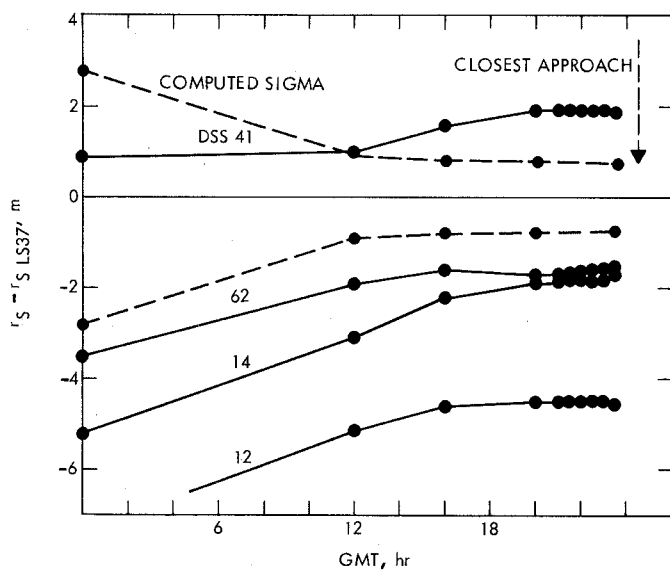


Fig. 14. Mariner 9 spin axis values on 11-13-71, uncalibrated Doppler plus range